



# PEER-ASSESSMENT IN MATHEMATICS AT NTNU

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*Maugesten & Lauvås reported on spectacular improvements in student performance after the introduction of a peer-assessment methodology in a class of approximately 100 mathematics teacher students at Østfold University College in 2003.*

*We wanted to investigate whether similar gains could be made at NTNU and also consider the effect of peer-assessment in a more controlled study. Here we report on a pilot study conducted during the Spring Term 2005, in which peer-assessment was tried out in one section (consisting of teacher students) of the course MA1103, Multidimensional Analysis.*

## INTRODUCTION

Maugesten (2005a) asked “How can one reduce the percentage of students flunking at a mathematics exam in teacher education from more than 50% to 15%, and without teachers and students being stressed, and without lowering the standards?” Let us see what led to the substantial improvements in student performance in a class of approximately 100 mathematics teacher students at Østfold University College in 2003. First, the obligatory mathematics course for teacher students was divided into three modules. More important, the national Quality Reform with its focus on closer student guidance and new forms of assessment inspired Maugesten & Lauvås (2004) to introduce some special measures: Level differentiation and peer-assessment. One out of six hours of weekly class-time was used for peer-assessment where the students graded each others homework, helped by the teacher. The students had to participate seriously in seven out of nine peer-assessment sessions to be admitted to the exam.

Peer-assessment is certainly not a new idea, and systematic studies on its efficiency have been conducted for at least 30 years, Zariski (1996). However, not many of these studies focus on mathematics (cf. Hart, 1999), and some authors have even claimed that the exact character of the subject renders peer-assessment superfluous. Furthermore many of the studies are from the Anglo-Saxon world where students are typically younger than in our context. As one of the main ideas with peer-assessment is to help students take more responsibility of their own work, this is certainly an issue to be reckoned with.

We wanted to investigate whether similar gains similar to those at Østfold could be made at NTNU and at the same time consider the effect of peer-assessment in a more



controlled study. Here we report on a pilot study conducted during the Spring Term 2005, in which peer-assessment was tried out in one section (consisting of teacher students) of the course MA1103, Multidimensional Analysis.

## OUR SETUP

The class MA1103 consisted of two two-hour lectures in a class of approximately 100 students, and a two-hour tutorial (problem session) with groups of 20–30 students. All students enrolled in the teacher-education program (LUR) were in the same tutorial group, in which a peer-assessment was used. The other tutorial groups followed the normal format, where teaching assistants graded the homework. For all students 8 out of 12 homework assignments had to be approved.

A typical LUR tutorial group started by dividing the students into pairs. The pairs swapped their written solutions of the home-work problems and corrected them, using a suggested solutions handout, the text-book and the teacher for help. The students were asked to give hints and feedback on the solutions they corrected. This lasted one class hour. In the second class hour, the teacher explained unclear problems on the blackboard.

As a basis for our analysis we collected the grades of all students who completed the course. We also surveyed student opinion on peer-assessment in the LUR tutorial group on two occasions, one early on in the course and the second in the study period after the end of the lectures.

## EXAM RESULTS

Out of the 100 students in MA 1103, some 70 also participated in the class MA 1202, Linear algebra and applications. In this sample the grades on the exams are as shown in Table 1.

| Grades in %        | MA1103 |    |    |    |    |    | MA1202 |   |    |    |    |    |
|--------------------|--------|----|----|----|----|----|--------|---|----|----|----|----|
|                    | A      | B  | C  | D  | E  | F  | A      | B | C  | D  | E  | F  |
| LUR ( $n = 21$ )   | 24     | 29 | 10 | 5  | 19 | 14 | 14     | 0 | 14 | 29 | 24 | 19 |
| Other ( $n = 47$ ) | 19     | 30 | 23 | 15 | 6  | 6  | 9      | 2 | 23 | 13 | 34 | 19 |

Table 1: Grades in percent for LUR-students and other students

The averages based on the passing grades  $A = 5$ ,  $B = 4$ ,  $C = 3$ ,  $D = 2$ ,  $E = 1$  are as follows. For MA1103 the LUR-group gets 3.39 and the other group 3.43. For MA1202 the averages are 2.41 and 2.24, respectively.

It is clear that there is no difference in average grades between the two groups of students. There appears to be a larger variance for the LUR students, especially in the MA1103 class, but a two-tail F-test of the variances shows that these are not statistically significant (smallest  $p$  is about 0.09).

The results from the analysis course in the fall, which is the foundation for MA1103, were also available. Here one of 21 LUR students got an F, and the average based on



the passing grades was 3.00. (In the linear algebra course preceding MA1202 none of the LUR students flunked and the average was 4.00!)

## **SURVEY RESULTS**

The February survey was answered by 19 of 21 LUR students. The complete survey, along with the results, is shown in Appendix A.

The May questionnaire was handled electronically and 11 of the 21 students answered. The questionnaire had many of the same statements as the first one. It is interesting to notice that the answer distribution to the statement “I read the teachers’ and tutors’ feedback carefully” now is 10-1-0-0-0. In the second part of the questionnaire we noted that the students agreed most to the statement “Teacher students should have at least one tutorial like this one” (0-1-2-1-7) and meant that “Other students than the teacher students should have at least part of a tutorial like this one” (0-1-2-6-2). However, to the statement “The description of the tutorial presented in the beginning of the term was adequate” the answers were 0-4-2-4-1.

## **DISCUSSION**

Our initial findings, although based on a rather small sample, are not in line with the large improvements reported by Maugesten & Lauvås (2004). Furthermore, they indicated that peer-assessment was most beneficial for the weakest students. In contrast to this, we found weak evidence on the opposite phenomenon, i.e. that peer-assessment would have had a polarizing effect. One possible explanation for this is the difficulty of the course: Maugesten & Lauvås (2004) reported on the first two modules of their mathematics curriculum. On the exam in the third module with more new and difficult material the students had a high percentage of failure (Maugsten, 2005b).

Peer-assessment presumably also has qualities not measured in an exam, like preparing students give adequate feedback. In our case it is clear that we should have explained and discussed the whole setup of peer-assessment more closely in the beginning. Two of the LUR students sum up the situation quite well in a comment on the questionnaire: “This way to arrange the problem sessions has a potential for success. But in the situation when the participants are poorly prepared for the task it is rather inefficient.”

In the next stage of the project we will try out the peer-assessment (somewhat revised based on the feedback from the students) in a larger class of engineering students as well as in MA1103 next spring.

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## APPENDIX A - THE FIRST SURVEY

| How do the following statements relate to you <b>last term</b> ?        | A  | B | C | D | E |
|---|----|---|---|---|---|
| I worked on homework problems with my classmates also out-of-class      | 14 | 1 | 2 | 1 | 1 |
| I always tried to work on all the homework problems                     | 8  | 8 | 2 | 1 | 0 |
| I focused on the most important homework problems                       | 11 | 5 | 2 | 1 | 0 |
| I often compared my solutions with other students' before handing it in | 10 | 3 | 4 | 2 | 0 |
| I read teachers' and tutors' feedback carefully                         | 7  | 4 | 2 | 0 | 6 |

A – This is what I did.

B – I would have liked to do this, but couldn't.

C – I did not think this was necessary.

D – I did not think this was possible.

E – I'll do it next time/later/some other time.

Please check one box that best describes your opinion on MA1103 now:

| Do you agree with the following statements?  | 1 | 2  | 3 | 4  | 5  |
|--|---|----|---|----|----|
| I enjoy working with mathematical problems   | 0 | 0  | 3 | 11 | 5  |
| I am among the strongest mathematically in the tutorial group                                    | 5 | 5  | 7 | 2  | 0  |
| I enjoy explaining mathematical ideas to others  | 0 | 0  | 6 | 10 | 3  |
| I spend a lot of time on mathematics homework<br># hours per week: $6.4 \pm 2.9$                 |   |    |   |    |    |
| I would spend more time on homework, if the benefit was greater                                  | 0 | 5  | 9 | 2  | 3  |
| Since other students see my work, I will work harder   | 0 | 2  | 5 | 10 | 2  |
| I understand the motivation for this type of tutorial  | 0 | 0  | 3 | 12 | 4  |
| It is a good idea to have a tutorial like this   | 0 | 0  | 3 | 9  | 7  |
| There should be more tutorials like this one   | 0 | 0  | 7 | 6  | 6  |
| Tutorials like this make it more difficult to have both fast and slow students in the same group | 3 | 10 | 2 | 3  | 1  |
| Working in groups is good for learning mathematics   | 0 | 0  | 1 | 11 | 7  |
| Working in groups is good for communicating and teaching mathematical ideas                      | 0 | 0  | 2 | 6  | 11 |
| Working in groups is more fun and inspiring  | 0 | 0  | 3 | 6  | 10 |
| Assessing other students' solutions deepens my understanding of the material                     | 0 | 0  | 5 | 9  | 5  |
| Assessing other students' solutions helps me write my own solutions clearer                      | 0 | 0  | 5 | 9  | 5  |

1 – definitely not    2 – no    3 – maybe, not applicable    4 – yes    5 -- definitely